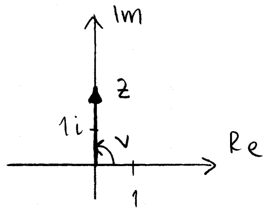
4229

(a) $z = 2i = 0 + 2i$

Ur figuren ser vi direkt att

$r = 2, \quad v = \frac{\pi}{2}$



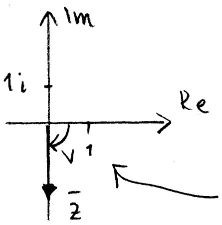
Alltså:

$$z = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \quad \text{(Svar)}$$

$\bar{z} = -2i = 0 - 2i$

Ur figuren ser vi direkt att

$r = 2, \quad v = -\frac{\pi}{2}$



Vrider vi medurs från reella axeln räknas vinkeln negativ.

Normalt hade vi valt $v = \frac{3\pi}{2}$, men här skulle ju argumentet vara i intervallet $-\pi$ till π.

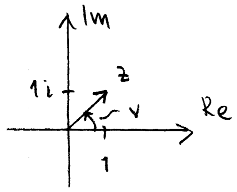
Alltså:

$$\bar{z} = 2\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right) \quad \text{(Svar)}$$

(b) $z = 1 + i$

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\tan v = \frac{1}{1} \Rightarrow v = \frac{\pi}{4}$



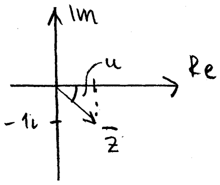
Alltså:

$$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) \quad \text{(Svar)}$$

$\bar{z} = 1 - i$

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\tan u = \frac{1}{1} \Rightarrow u = \frac{\pi}{4}.$



Då är $\arg \bar{z} = -\frac{\pi}{4}$.

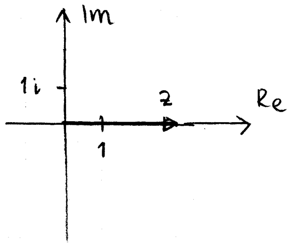
Alltså

$$\bar{z} = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right) \quad \text{(Svar)}$$

4229 (forts)

(c) $z = 3 = 3 + 0i$

Ur figuren ser vi direkt att



$r = 3, \quad v = 0$

Alltså

$z = 3(\cos 0 + i \sin 0)$ (Svar)

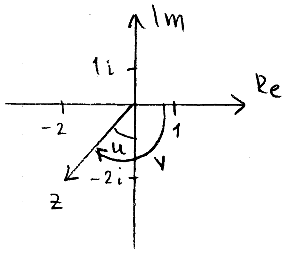
$\bar{z} = 3. \quad (= 3 - 0i)$

Ett reellt tal har sig självt som konjugat

Alltså

$\bar{z} = 3(\cos 0 + i \sin 0)$ (Svar)

(d) $z = -2 - 2i$



$r = \sqrt{2^2 + 2^2} = \sqrt{8}$

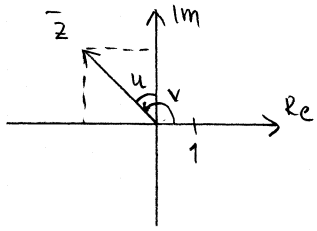
$\tan u = \dfrac{2}{2} \Rightarrow u = \dfrac{\pi}{4}$

Då får vi $v = -\dfrac{\pi}{2} - \dfrac{\pi}{4} = -\dfrac{3\pi}{4}$

Alltså

$z = \sqrt{8}\left(\cos\left(-\dfrac{3\pi}{4}\right) + i \sin\left(-\dfrac{3\pi}{4}\right)\right)$ (Svar)

$\bar{z} = -2 + 2i$



$r = \sqrt{2^2 + 2^2} = \sqrt{8}$

$\tan u = \dfrac{2}{2} \Rightarrow u = \dfrac{\pi}{4}.$

Då får vi $v = \dfrac{\pi}{2} + \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

Alltså: $\bar{z} = \sqrt{8}\left(\cos\left(\dfrac{3\pi}{4}\right) + i \sin\left(\dfrac{3\pi}{4}\right)\right)$ (Svar)