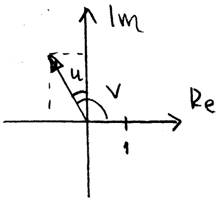
$\boxed{4312}$ (a) Gå över till polär form:

$(-1+\sqrt{3}i) =$

$r = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{4} = 2$

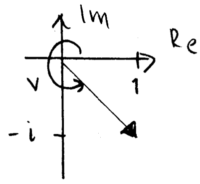


$\tan u = \dfrac{1}{\sqrt{3}} \Rightarrow u = \dfrac{\pi}{6}$

Då är $v = \dfrac{\pi}{2} + \dfrac{\pi}{6} = \dfrac{4\pi}{6} = \dfrac{2\pi}{3}$

$= 2\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$

$(1-i) =$



Vi ser direkt att $r = \sqrt{2}$ och att

$v = \dfrac{7\pi}{4}$

$= \sqrt{2}\left(\cos\dfrac{7\pi}{4} + i\sin\dfrac{7\pi}{4}\right)$

Då får vi

deMoivres formel

$$z = \frac{\left(2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\right)^6}{\left(\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\right)^9} = \frac{2^6\left(\cos\frac{6\cdot 2\pi}{3} + i\sin\frac{6\cdot 2\pi}{3}\right)}{\sqrt{2}\cdot(\sqrt{2})^8\left(\cos\frac{9\cdot 7\pi}{4} + i\sin\frac{9\cdot 7\pi}{4}\right)}$$

$(\sqrt{2})^9 = \sqrt{2}\cdot(\sqrt{2})^8$

$\cos 4\pi = \cos(4\pi - 2\cdot 2\pi)$
$= \cos 0$

$\cos\dfrac{63\pi}{4} =$
$= \cos\left(\dfrac{63\pi}{4} - 7\cdot 2\pi\right)$
$= \cos\dfrac{7\pi}{4}$

$$= \frac{2^6(\cos 4\pi + i\sin 4\pi)}{\sqrt{2}\cdot 2^4\left(\cos\frac{63\pi}{4} + i\sin\frac{63\pi}{4}\right)} = \frac{2^2(\cos 0 + i\sin 0)}{\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)}$$

$\dfrac{4}{\sqrt{2}} = \dfrac{\sqrt{16}}{\sqrt{2}} = \sqrt{\dfrac{16}{2}} = \sqrt{8}$

$\boxed{4312}$

(forts)

$$= \sqrt{8}\left(\cos\left(0-\frac{7\pi}{4}\right)+i\sin\left(0-\frac{7\pi}{4}\right)\right)$$

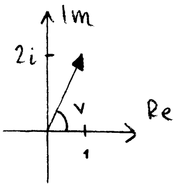
$$= \sqrt{8}\left(\cos\left(-\frac{7\pi}{4}\right)+i\sin\left(-\frac{7\pi}{4}\right)\right) = \sqrt{8}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$$

$\cos\left(-\frac{7\pi}{4}+1\cdot 2\pi\right) = \cos\frac{\pi}{4}$

Svar: Absolutbeloppet $\sqrt{8}$, argumentet $\frac{\pi}{4}$.

(b) Gå över till polär form:

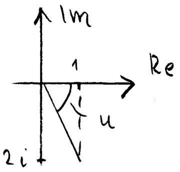
$(1+2i) = \Big\{$ $r=\sqrt{1^2+2^2}=\sqrt{5}$



$v = \arctan 2$ $\Big\}$ $= \sqrt{5}\left(\cos(\arctan 2)+i\sin(\arctan 2)\right)$

arctan 2 ≈ 1,1071, men vi fortsätter exakt

$(1-2i) = \Big\{$ $r=\sqrt{1^2+2^2}=\sqrt{5}$



$u = \arctan 2$

Då är argumentet $-\arctan 2$ $\Big\}$ $\sqrt{5}\left(\cos(-\arctan 2)+i\sin(-\arctan 2)\right)$

Då får vi

$$z = \frac{\left(\sqrt{5}\left(\cos(\arctan 2)+i\sin(\arctan 2)\right)\right)^{12}}{\left(\sqrt{5}\left(\cos(-\arctan 2)+i\sin(-\arctan 2)\right)\right)^{9}} =$$

$$= \frac{(\sqrt{5})^{12}\left(\cos(12\arctan 2)+i\sin(12\arctan 2)\right)}{(\sqrt{5})^{9}\left(\cos(-9\arctan 2)+i\sin(-9\arctan 2)\right)}$$

$\boxed{4312}$

(forts)

$$= (\sqrt{5})^3 \Big(\cos(12 \arctan 2 - (-9 \arctan 2) + i \sin(12 \arctan 2 - (-9 \arctan 2))\Big)$$

$(\sqrt{5})^3 = (\sqrt{5})^2 \cdot \sqrt{5} = 5 \cdot \sqrt{5}$

$$= 5\sqrt{5}\,\big(\cos(21 \arctan 2) + i \sin(21 \arctan 2)\big)$$

$21 \arctan 2$
$\approx 23{,}250$

$$= 5\sqrt{5}\,\big(\cos(21 \arctan 2 - 3 \cdot 2\pi) + i \sin(21 \arctan 2 - 3 \cdot 2\pi)\big)$$

Svar: Absolutbeloppet $5\sqrt{5}$, argumentet $21 \arctan 2 - 6\pi$ $(\approx 4{,}401)$