$\boxed{32}$

Bl:övn 4.

Uppgiften är egentligen att hitta samtliga rötter till ekvationen

$$z^3 = \sqrt{32} + \sqrt{32}\,i \qquad (*)$$

Ansätt $z = r(\cos v + i \sin v)$

Då är $z^3 = r^3(\cos 3v + i \sin 3v)$

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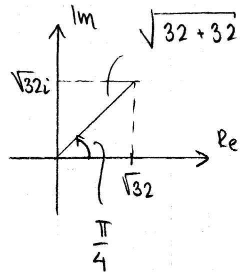
Skriv $\sqrt{32} + \sqrt{32}\, i$ på polär form:

$$\sqrt{32} + \sqrt{32}\, i = 8\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$$




$\sqrt{32+32} = \sqrt{64} = 8$

Då kan vi skriva ekvationen $(*)$ som

$$r^3(\cos 3v + i \sin 3v) = 8\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$$

VL = HL om

$$r^3 = 8 \qquad \text{och} \qquad 3v = \frac{\pi}{4} + n \cdot 2\pi$$

$$r = 2$$

$$v = \frac{\pi}{12} + n\frac{2\pi}{3}, \quad n \text{ heltal}$$

Nu kan vi göra en lista med rötterna till $(*)$:

$n = 0$: $\quad v = \dfrac{\pi}{12} \qquad \underline{\underline{z_1 = 2\left(\cos\dfrac{\pi}{12} + i \sin\dfrac{\pi}{12}\right)}}$

$n = 1$: $\quad v = \dfrac{\pi}{12} + \dfrac{2\pi}{3} = \dfrac{\pi}{12} + \dfrac{8\pi}{12} = \dfrac{9\pi}{12} = \dfrac{3\pi}{4}$

$$\underline{\underline{z_2 = 2\left(\cos\frac{3\pi}{4} + i \sin\frac{3\pi}{4}\right)}}$$

$n = 2$: $\quad v = \dfrac{\pi}{12} + 2 \cdot \dfrac{2\pi}{3} = \dfrac{\pi}{12} + \dfrac{4\pi}{3} = \dfrac{\pi}{12} + \dfrac{16\pi}{12} = \dfrac{17\pi}{12}$

$$\underline{\underline{z_3 = 2\left(\cos\frac{17\pi}{12} + i \sin\frac{17\pi}{12}\right)}}$$