7

$$\int_1^e \frac{1}{x}\,dx = \Big[\ln x\Big]_1^e = \underbrace{\ln e}_{1} - \underbrace{\ln 1}_{0} = 1 - 0 = 1$$

Svar: Kerstin har gjort rätt (se korrekt beräkning ovan)

8

$z_1 \cdot z_2 = 7 + i$ ger

$$z_2 = \frac{7+i}{z_1} = \{ z_1 = 3 - i \text{ enligt uppg} \} = \frac{7+i}{3-i} = \frac{(7+i)(3+i)}{(3-i)(3+i)}$$

$$= \frac{21 + 7i + 3i + i^2}{9 - i^2} = \{ i^2 = -1 \} = \frac{20 + 10i}{10} = 2 + i$$

Svar: $z_2 = 2 + i$

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9 (a)

Visa att $\cos^2 x \left(\dfrac{\sin^2 x}{\cos^2 x} + 1 \right) = 1$

Trig. ettan

$$VL = \cos^2 x \left(\frac{\sin^2 x}{\cos^2 x} + 1 \right) = \frac{\cancel{\cos^2 x}\,\sin^2 x}{\cancel{\cos^2 x}} + \cos^2 x = \sin^2 x + \cos^2 x = 1 = HL$$

□

(b)

Visa att $\sqrt{2}\cos\left(x + \dfrac{\pi}{4}\right) = \cos x - \sin x$

Additionssats för cosinus

$$VL = \sqrt{2}\cos\left(x + \frac{\pi}{4}\right) = \sqrt{2}\left(\cos x \cdot \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4}\right)$$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$= \sqrt{2}\left(\cos x \cdot \frac{1}{\sqrt{2}} - \sin x \cdot \frac{1}{\sqrt{2}}\right)$$

$$= \cos x - \sin x = HL$$ □
