$y = f(u)$ där $u = g(x)$ har... ...derivatan $y' = f'(u) \cdot u'$

$$\frac{dy}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

<u>Uppvärmning:</u>

① $y = (\sin x)^2$

$y = u^2$ där $u = \sin x$

$y' = 2u \cdot u'$
$= 2 \sin x \cdot \cos x$

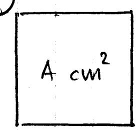
Egentligen: $y'(x) = 2u(x) \cdot u'(x)$

② $y = (u(x))^2$

$[\, y(u(x)) = (u(x))^2 \,]$

$y'(x) = 2u \cdot u'(x)$

$\boxed{3172}$ (a)

① 

s = 12 cm

③ $A = s^2$

Samband mellan variablerna

$A(t) = (s(t))^2$

② Bestäm $\frac{dA}{dt}$ om $\frac{ds}{dt} = 1{,}5$ (cm/min) då $s = 12$ (cm)

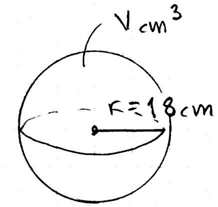
Kedjeregeln:

Samband mellan förändringshastigheterna

④ $$\boxed{\frac{dA}{dt}} = \frac{dA}{ds} \cdot \frac{ds}{dt} = \boxed{2s \cdot \frac{ds}{dt}}$$

$$= 2 \cdot 12 \cdot 1{,}5 = 36 \ (\text{cm}^2/\text{min})$$

$\boxed{3173}$



$$V = \frac{4\pi r^3}{3}$$

$$V(r(t)) = \frac{4\pi (r(t))^3}{3}$$

Bestäm $\frac{dr}{dt}$ om $\frac{dV}{dt} = 30$ (cm³/s) då $r = 18$ cm

Kedjeregeln:

$$\boxed{\frac{dV}{dt}} = \frac{dV}{dr} \cdot \frac{dr}{dt} = \frac{4\pi \cdot 3r^2}{3} \cdot \frac{dr}{dt} = \boxed{4\pi r^2 \cdot \frac{dr}{dt}} \quad (*)$$

Insättning i $(*)$ ger:

$$30 = 4\pi \cdot 18^2 \cdot \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = 0{,}0074 \ (\text{cm/s})$$