

4253

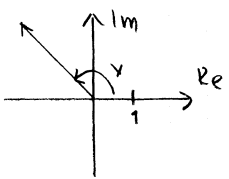
Vi förenklar först:

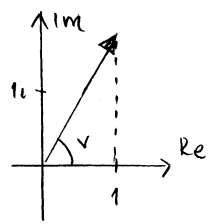
Förläng med  $(1+i)$ 

$$\frac{i}{1 + \frac{i(\sqrt{3}-1)}{1+i}} \stackrel{\text{Förläng med } (1+i)}{=} \frac{i(1+i)}{\left(1 + \frac{i(\sqrt{3}-1)}{1+i}\right)(1+i)} = \frac{i-1}{1+i+i\sqrt{3}-i}$$

$$= \frac{i-1}{1+i\sqrt{3}} = \frac{-1+i}{1+i\sqrt{3}}$$

Här stoppar vi och går över till polar form:

$$-1+i = \left\{ \begin{array}{l} r = \sqrt{1^2+1^2} = \sqrt{2} \\ \varphi = \frac{3\pi}{4} \text{ (ses i figur)} \end{array} \right\} = \sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$


$$1+i\sqrt{3} = \left\{ \begin{array}{l} r = \sqrt{1+(\sqrt{3})^2} = \sqrt{4} = 2 \\ \varphi = \frac{\pi}{3} \end{array} \right\} = 2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$


$$\tan \varphi = \frac{\sqrt{3}}{1} \Rightarrow \varphi = \frac{\pi}{3}$$

Då får vi

$$\frac{-1+i}{1+i\sqrt{3}} = \frac{\sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)}{2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)}$$

och, det sökta argumentet är

$$\frac{3\pi}{4} - \frac{\pi}{3} = \frac{9\pi}{12} - \frac{4\pi}{12} = \frac{5\pi}{12}$$

Vid division subtraheras argumenten!

$$\underline{\underline{\text{Svar: } \frac{5\pi}{12}}}$$