NP Ma 4 vt 2014

$\boxed{18}$

$f(x) = -x\ln x\,, \quad x > 0$

Derivatans nollställen?

$$f'(x) = -\left(1 \cdot \ln x + x \cdot \frac{1}{x}\right) = -(\ln x + 1) = -1 - \ln x$$

$$f'(x) = 0 \text{ ger } -1 - \ln x = 0$$

$$\ln x = -1$$

$$x = e^{-1} \quad \left(= \frac{1}{e}\right)$$

Hade också kunnat fortsätta med andraderivata

Teckentabell

x	(0)		$\frac{1}{e}$	
$f'(x)$		$+$	0	$-$
$f(x)$		$\nearrow$	MAX	$\searrow$

$$f'\left(\frac{1}{e^2}\right) = -1 - \ln \underbrace{e^{-2}}_{=e^{-2}}$$
$$= -1 - (-2)\underbrace{\ln e}_{=1} = 1 > 0$$

$$f'(e) = -1 - \ln e = -1 - 1$$
$$= -2 < 0$$

Extremvärden

$$f\left(\frac{1}{e}\right) = -\frac{1}{e} \ln\frac{1}{e} = -\frac{1}{e} \ln e^{-1} = -\frac{1}{e}(-1)\cdot\underbrace{\ln e}_{=1} = \frac{1}{e}$$

Svar: Max i $\left(\frac{1}{e}, \frac{1}{e}\right)$