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Gå över till polär form:

$$(2+2i) = \left\{ \begin{array}{l} r = \sqrt{2^2 + 2^2} = \sqrt{8} \\ \varphi = \frac{\pi}{4} \text{ (ses direkt i figur)} \\ \begin{array}{c} \text{Im} \\ \uparrow \\ 1i \\ \swarrow \text{ } 2+2i \\ \searrow \text{ } \\ \downarrow \text{ } \\ \text{Re} \\ \leftarrow 1 \end{array} \end{array} \right\} = \sqrt{8} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$(1+\sqrt{3})i = \left\{ \begin{array}{l} r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2 \\ \tan \varphi = \frac{\sqrt{3}}{1} \Rightarrow \varphi = \frac{\pi}{3} \\ \begin{array}{c} \text{Im} \\ \uparrow \\ \sqrt{3} \\ \swarrow \text{ } 1+\sqrt{3}i \\ \searrow \text{ } \\ \downarrow \text{ } \\ \text{Re} \\ \leftarrow 1 \end{array} \end{array} \right\} = 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$3i = \left\{ \begin{array}{l} \begin{array}{c} \text{Im} \\ \uparrow \\ \text{ } \\ \downarrow \text{ } \\ \text{Re} \\ \leftarrow \end{array} \\ r = 3 \\ \varphi = \frac{\pi}{2} \end{array} \right\} = 3 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$(\sqrt{12} - 2i) = \left\{ \begin{array}{l} r = \sqrt{(\sqrt{12})^2 + 2^2} = \sqrt{16} = 4 \\ \begin{array}{c} \text{Im} \\ \uparrow \\ \text{ } \\ \downarrow \text{ } \\ \text{Re} \\ \leftarrow \sqrt{12} \end{array} \\ \begin{array}{c} \text{ } \\ \swarrow \text{ } \\ \searrow \text{ } \\ \downarrow \text{ } \\ \text{ } \end{array} \end{array} \right\} = 4 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right)$$

$\tan u = \frac{2}{\sqrt{12}} = \frac{2}{\sqrt{4 \cdot 3}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$
 $\Rightarrow u = \frac{\pi}{6}$
 Da får vi
 $\varphi = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

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(forts)

Talet i uppgiften kan alltså skrivas

$$\frac{\sqrt{8} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \cdot 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)}{3 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) \cdot 4 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right)}$$

Talets absolut belopp är

$$\frac{\sqrt{8} \cdot \cancel{2}}{3 \cdot \frac{4}{2}} = \frac{\sqrt{4 \cdot 2}}{3 \cdot 2} = \frac{\sqrt{4} \cdot \sqrt{2}}{3 \cdot 2} = \frac{\cancel{2} \cdot \sqrt{2}}{3 \cdot 2} = \frac{\sqrt{2}}{3}$$

och argumentet är

$$\frac{\pi}{4} + \frac{\pi}{3} - \left(\frac{\pi}{2} + \frac{11\pi}{6} \right) = \frac{3\pi}{12} + \frac{4\pi}{12} - \frac{6\pi}{12} - \frac{22\pi}{12} = -\frac{21\pi}{12}$$

Vi adderar $2\pi = \frac{24\pi}{12}$ för att få argumentet i intervallet 0 till 2π :

$$-\frac{21\pi}{12} + \frac{24\pi}{12} = \frac{3\pi}{12} = \frac{\pi}{4}$$

Talet kan alltså till sist skrivas

$$\frac{\sqrt{2}}{3} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \quad (\underline{\underline{\text{Svar}}})$$

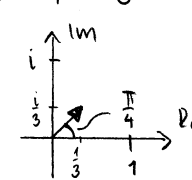
Om vi först förenklar och sedan går över till polär form blir det så här:

$$\frac{(2+2i)(1+\sqrt{3}i)}{3i(\sqrt{12}-2i)} = \frac{2(1+i)(1+\sqrt{3}i)}{3i(2\sqrt{3}-2i)} = \frac{\cancel{2}(1+i)(1+\sqrt{3}i)}{3i \cdot \cancel{2}(\sqrt{3}-i)}$$

$$\begin{aligned} \sqrt{12} &= \sqrt{4 \cdot 3} = \\ &= \sqrt{4} \cdot \sqrt{3} = 2 \cdot \sqrt{3} \end{aligned}$$

$$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$= \frac{(1+i)(1+\sqrt{3}i)}{3(1+\sqrt{3}i)} = \frac{1+i}{3} = \frac{1}{3} + \frac{1}{3}i =$$

$$\left. \begin{aligned} r &= \sqrt{\frac{1}{9} + \frac{1}{9}} = \frac{\sqrt{2}}{3} \\ \varphi &= \frac{\pi}{4} \end{aligned} \right\}$$


$$= \frac{\sqrt{2}}{3} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \quad (\underline{\underline{\text{Svar}}})$$

Här gick delta mkt fortare. Dock inte så lätt att på för hand avgöra vilken väg som är snabbast.