

4311

(a) Gå över till polär form:

$$(-1 + \sqrt{3}i) = \left\{ \begin{array}{l} r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2 \\ \begin{array}{c} \text{Im} \\ \uparrow \\ \text{Re} \end{array} \\ \begin{array}{c} \text{u} \\ \swarrow \\ \text{v} \end{array} \\ \begin{array}{c} 1 \\ \downarrow \\ 1 \end{array} \end{array} \right\} = 2 \left( \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

$$\tan u = \frac{1}{\sqrt{3}} \Rightarrow u = \frac{\pi}{6}$$

$$\text{Då är } v = \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$(1 - i) = \left\{ \begin{array}{l} \begin{array}{c} \text{Im} \\ \uparrow \\ \text{Re} \end{array} \\ \begin{array}{c} \text{v} \\ \downarrow \\ -i \end{array} \\ \begin{array}{c} 1 \\ \downarrow \\ 1 \end{array} \end{array} \right\} = \sqrt{2} \left( \cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$

Ni ser direkt att  $r = \sqrt{2}$  och att

$$v = \frac{7\pi}{4}$$

Då får vi

$$z = \frac{\left( 2 \left( \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) \right)^6}{\left( \sqrt{2} \left( \cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right) \right)^9}$$

de Moivre's formel

$$= \frac{2^6 \left( \cos \frac{6 \cdot 2\pi}{3} + i \sin \frac{6 \cdot 2\pi}{3} \right)}{\sqrt{2} \cdot (\sqrt{2})^8 \left( \cos \frac{9 \cdot 7\pi}{4} + i \sin \frac{9 \cdot 7\pi}{4} \right)}$$

$$(\sqrt{2})^9 = \sqrt{2} \cdot (\sqrt{2})^8$$

$$\cos 4\pi = \cos(4\pi - 2 \cdot 2\pi) = \cos 0$$

$$\cos \frac{63\pi}{4} =$$

$$= \cos \left( \frac{63\pi}{4} - 7 \cdot 2\pi \right)$$

$$= \cos \frac{7\pi}{4}$$

$$= \frac{2^6 (\cos 4\pi + i \sin 4\pi)}{\sqrt{2} \cdot 2^4 \left( \cos \frac{63\pi}{4} + i \sin \frac{63\pi}{4} \right)} = \frac{2^2 (\cos 0 + i \sin 0)}{\sqrt{2} \left( \cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)}$$

$$\frac{4}{\sqrt{2}} = \frac{\sqrt{16}}{\sqrt{2}} = \sqrt{\frac{16}{2}} = \sqrt{8}$$

4311

(forts)

$$= \sqrt{8} \left( \cos \left( 0 - \frac{7\pi}{4} \right) + i \sin \left( 0 - \frac{7\pi}{4} \right) \right)$$

$$= \sqrt{8} \left( \cos \left( -\frac{7\pi}{4} \right) + i \sin \left( -\frac{7\pi}{4} \right) \right) = \sqrt{8} \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$\cos \left( -\frac{7\pi}{4} + 1 \cdot 2\pi \right) = \cos \frac{\pi}{4}$$

Svar: Absolutbeloppet  $\sqrt{8}$ , argumentet  $\frac{\pi}{4}$ .

(b) Gå över till polär form:

$$(1+2i) = \left\{ \begin{array}{l} r = \sqrt{1^2 + 2^2} = \sqrt{5} \\ \begin{array}{c} \text{Im} \\ 2i \\ \nearrow \\ \text{Re} \\ 1 \end{array} \\ \varphi = \arctan 2 \end{array} \right\} = \sqrt{5} \left( \cos(\arctan 2) + i \sin(\arctan 2) \right)$$

arctan 2  
 $\approx 1,1071$ ,  
 men vi fortsätter  
 exakt

$$(1-2i) = \left\{ \begin{array}{l} r = \sqrt{1^2 + 2^2} = \sqrt{5} \\ \begin{array}{c} \text{Im} \\ 1 \\ \searrow \\ \text{Re} \\ 1 \end{array} \\ -2i \\ \varphi = \arctan 2 \\ \text{Då är argumentet } -\arctan 2 \end{array} \right\} = \sqrt{5} \left( \cos(-\arctan 2) + i \sin(-\arctan 2) \right)$$

Då får vi

$$\begin{aligned} z &= \frac{\left( \sqrt{5} \left( \cos(\arctan 2) + i \sin(\arctan 2) \right) \right)^{12}}{\left( \sqrt{5} \left( \cos(-\arctan 2) + i \sin(-\arctan 2) \right) \right)^9} = \\ &= \frac{(\sqrt{5})^{12} \left( \cos(12 \arctan 2) + i \sin(12 \arctan 2) \right)}{(\sqrt{5})^9 \left( \cos(-9 \arctan 2) + i \sin(-9 \arctan 2) \right)} \end{aligned}$$

4311

(forts)

$$(\sqrt{5})^3 = (\sqrt{5})^2 \cdot \sqrt{5} = 5\sqrt{5}$$

$$21 \arctan 2 \\ \approx 23,250$$

$$= (\sqrt{5})^3 \left( \cos(12 \arctan 2 - (-9 \arctan 2)) \right. \\ \left. + i \sin(12 \arctan 2 - (-9 \arctan 2)) \right)$$

$$= 5\sqrt{5} \left( \cos(21 \arctan 2) + i \sin(21 \arctan 2) \right)$$

$$= 5\sqrt{5} \left( \cos(21 \arctan 2 - 3 \cdot 2\pi) + i \sin(21 \arctan 2 - 3 \cdot 2\pi) \right)$$

Svar: Absolutbeloppet  $5\sqrt{5}$ , argumentet  $21 \arctan 2 - 6\pi$  ( $\approx 4,401$ )