

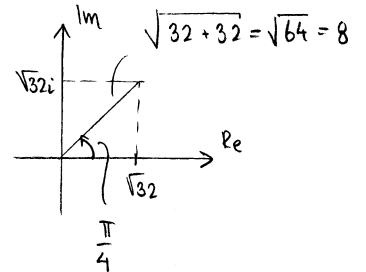
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Bl.öv 4

Uppgiften är egentligen att hitta samtliga rötter till ekvationen

$$z^3 = \sqrt{32} + \sqrt{32}i \quad (*)$$

$$\left. \begin{array}{l} \text{Ansätt } z = r(\cos v + i \sin v) \\ \text{Då är } z^3 = r^3(\cos 3v + i \sin 3v) \\ \text{-----} \\ \text{Skriv } \sqrt{32} + \sqrt{32}i \text{ på polar form:} \\ \sqrt{32} + \sqrt{32}i = 8(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}) \end{array} \right\}$$



Då kan vi skriva ekvationen (\*) som

$$r^3(\cos 3v + i \sin 3v) = 8(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$$

VL = HL om

$$r^3 = 8 \quad \text{och} \quad 3v = \frac{\pi}{4} + n \cdot 2\pi$$

$$r = 2$$

$$v = \frac{\pi}{12} + n \cdot \frac{2\pi}{3}, \quad n \text{ heltal}$$

Nu kan vi göra en lista med rötterna till (\*):

$$n=0: \quad v = \frac{\pi}{12} \quad \underline{\underline{z_1 = 2 \left( \cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)}}$$

$$n=1: \quad v = \frac{\pi}{12} + \frac{2\pi}{3} = \frac{\pi}{12} + \frac{8\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

$$\underline{\underline{z_2 = 2 \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)}}$$

$$n=2: \quad v = \frac{\pi}{12} + 2 \cdot \frac{2\pi}{3} = \frac{\pi}{12} + \frac{4\pi}{3} = \frac{\pi}{12} + \frac{16\pi}{12} = \frac{17\pi}{12}$$

$$\underline{\underline{z_3 = 2 \left( \cos \frac{17\pi}{12} + i \sin \frac{17\pi}{12} \right)}}$$